Draft for fuzzy ternary logic.

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Example: a OR b, with a,b > 0. Want the following criteria to be true (obviously something can't be more than 100% certain.)

$$\|\xi^T \xi\| = \|\sigma^T \sigma\| = 1$$

So, the matrices look like the following

$$\xi = \begin{pmatrix} a \\ \sqrt{1-a^2} \end{pmatrix}, \sigma = \begin{pmatrix} b \\ \sqrt{1-b^2} \end{pmatrix}$$

For the simple case with values only in the realm of [0, 1], set up an operation matrix

$$O = N \begin{pmatrix} U \bigvee U & U \bigvee T \\ T \bigvee U & T \bigvee T \end{pmatrix}$$

Again, we want

$$\|O\| = 1$$

So,

$$N = \frac{1}{norm(O)}$$

$$X = \xi^T O \sigma = N \begin{pmatrix} a & \sqrt{1-a^2} \end{pmatrix} \begin{pmatrix} T \bigvee T & U \bigvee T \\ T \bigvee U & U \bigvee U \end{pmatrix} \begin{pmatrix} b \\ \sqrt{1-b^2} \end{pmatrix} =$$

$$= ... = N(abT \bigvee T + \sqrt{1-a^2}\sqrt{1-b^2} U \bigvee U + (b\sqrt{1-a^2} + a\sqrt{1-b^2}) U \bigvee T)$$

So, 0.3 OR 0.9 would be 0.915.

1 How about mixed true/false?

Indeed, how about -0.3 or 0.9?

Attach truth values to xi and sigma instead of O.

$$\xi = \begin{pmatrix} Fa \\ Ub \end{pmatrix}, \sigma = \begin{pmatrix} Tc \\ Ud \end{pmatrix}$$

$$O = N \begin{pmatrix} \bigvee & \bigvee \\ \bigvee & \bigvee \end{pmatrix}$$

$$X = N \begin{pmatrix} Fa & Ub \end{pmatrix} \begin{pmatrix} \bigvee & \bigvee \\ \bigvee & \bigvee \end{pmatrix} \begin{pmatrix} Tc \\ Ud \end{pmatrix} =$$

$$= N \begin{pmatrix} aF \bigvee + Ub \bigvee & aF \bigvee + Ub \bigvee \end{pmatrix} \begin{pmatrix} Tc \\ Ud \end{pmatrix}$$

$$N(acF \bigvee T + bcU \bigvee T + adF \bigvee U + dbU \bigvee U)$$

Reconstructing O

$$O = N \begin{pmatrix} F \bigvee T & U \bigvee T \\ F \bigvee U & U \bigvee U \end{pmatrix}$$

N is found to be

$$N = \frac{1}{\sqrt{2}}$$

Adding it all up, -0.3 OR 0.9 is found to be roughly 0.416